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INTEGRATOR FOR THE MAGNETIC-FIELD MEASUREMENT

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The search coil and integrator method for measuring the magnetic field has been widely adopted because of its high linearity and repeatability. The search coil has been analyzed in the Laslett Paper.[1] Here we will only study the behavior and characteristics of an electronic integrator.

In our application, the magnetic field ranging up to 22 kG is a slow time varying function. Errors due to the non-ideal characteristics of components are negligibly small. However, in measuring the field gradient, either a very precise measurement of B or a very high gain integrator is required because the $\frac{d(\Delta B)}{dt}$ signal is very small. In that case, those errors should be taken into account.

I. OPERATIONAL AMPLIFIER

An operational amplifier is the essential part of an integrator. An ideal operational amplifier has an infinite open loop gain over an infinite band width. The closed-loop equation (Fig. 1) can be written as:

$$e_o = -\frac{Z_f}{Z_i} e_i ,$$

where

e_o = output voltage

e_i = input voltage

Z_f = feedback impedance

Z_i = input impedance.

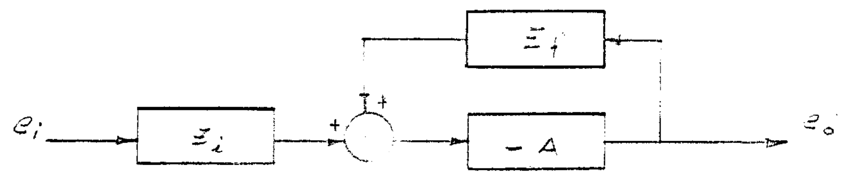


Fig. 1.

Practically all operational amplifiers have finite open-loop gain and band width. The gain and band-width product usually determines the cost of an operational amplifier. The transfer function[2] in the Laplace form of the Analog Device Model 210 operational amplifier which we first used can be approximately written as:

$$T_o(s) = \frac{-5.56 \times 10^6 \ (s+314)}{(s+0.6) \ (s+9400)} ,$$

where $S = \sigma + jw$; σ, w are real numbers.

S is a complex variable which is converted from the Laplace transformation $L\,[f(t)] = \int_0^\infty f(t)\, \epsilon^{-st}\, dt = F(s)$.

Some other types of operational amplifiers can be generalized as single pole or double-pole characteristics. The transfer function of a single-pole operational amplifier has the form:

$$T_o(s) = \frac{-A}{S + a} .$$

In that case, the closed-loop equation will be:

$$\frac{e_o}{e_i} = - \frac{Z_f}{Z_f} \left[\frac{1}{1 + \frac{1}{T_o(s)} \left(1 + \frac{Z_f}{Z_i} + \frac{Z_f}{Z_e} \right)} \right] ,$$

where Z_e is the input impedance of the operational amplifier.

If we treat the overall transfer function as two separate blocks connected, that is:

$$T_1(s) = - \frac{Z_f}{Z_i}$$

$$T_2(s) = \frac{1}{1 + \left(1 + \frac{Z_f}{Z_i} + \frac{Z_f}{Z_e} \right) / T_o(s)} ,$$

then $$T(s) = T_1(s) \cdot T_2(s).$$

$T_2(s)$ is an undesirable or error term. Bode diagrams (gain plot and phase curve) of T_2 will indicate all characteristics of an actual feedback amplifier from an ideal one. For an integrator, we have the following parameters:

$$Z_f = 1/SC$$

$$Z_i = R_i$$

$$Z_e = R_e$$

and $$T_o(s) = \frac{-A}{s + a}$$

For $R_i << R_e$

then $$T_2(s) = \frac{AS}{AS - (S + a)\ (S + 1/CR_1)}$$

or $$T_2(s) = \frac{-AS}{(S - A_1)\ (S - A_2)}$$

where $$A_1 A_2 = \frac{a}{CR_i}$$

$$A_1 + A_2 = A - a - \frac{1}{CR}$$

assuming $A_1 < A_2$.

On the B-t or ΔB-t curve (Fig. 2), A_1 affects the overall shape of the curve and A_2 affects only the bending points and the high frequency ripple on the curve.

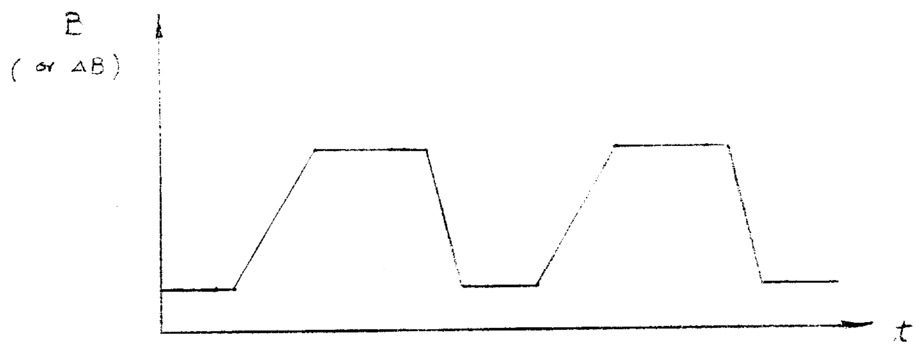


Fig. 2.

II. THE DRIFT OF AN INTEGRATOR

For a precision measurement, if the input signal is very small, the drift of an integrator introduces a considerable amount of error. A compensating circuit can be derived by first examining all effects in the circuit. It can also be compensated through a software program if a computer system is used. All the numerical data used here are from the circuit, 1 ms constant or R_i of 1 kohm and C of 1 μf, which we mostly used for the magnetic-field measurement of 3-foot model bending and quadrupole magnets.

1. The temperature effect of a charged capacitor. Due to the temperature change the voltage of a charged capacitor will be changed. This can be calculated by the following equations:

$$Q = CV$$

$$\frac{dV}{dT} = -\frac{Q}{C^2} \cdot \frac{dC}{dT} = -\frac{V}{C} \cdot \frac{dC}{dT}$$

where Q is the charge of the capacitor

C is the capacitance

V is the voltage

T is the temperature.

Suppose a capacitor has a temperature coefficient of -100 ppm/°C and $C = 1\mu f$ $V = 10^V$, then $\frac{dV}{dT} = -10^{-3}$ volt/°C.

This change of capacitance also affects the integrator RC constant, but by matching it with a positive temperature coefficient resistor we can eliminate this problem to a minimum.

2. The leakage resistance effect of a capacitor. There are many different types of capacitors. Among them, polystyrene capacitors are most suitable for our application. Other than having the lowest dielectric absorption, lowest temperature coefficient and highest stability, they also have the highest insulation resistance, usually 10^6 megaohm for a $1\mu f$ capacitor. The discharge rate due to the leakage resistance is

$$\frac{de_o}{dt} = \frac{-e_o}{CR} .$$

For $V = 10^V$, $CR = 10^6$ f - ohm

$$\left[\frac{de_o}{dt} \right]_{max} = -10^{-5} \, V/sec.$$

This factor is negligibly small. Due to the fact that the product of the insulation resistance and capacitance are constant, $\frac{de_o}{dt}$ is almost independent of C.

3. <u>The offset of the operational amplifier</u>. An integrator (Fig. 3) has an operational amplifier with a finite gain A. Usually R_e is much greater than R_i and R_o (output impedance). Then

$$\frac{dQ}{dt} = -\frac{e_e}{R_i}$$

$$\frac{de_o}{dt} = -\frac{e_o}{CR_iA} .$$

Assuming R_i = 1 kohm C = 1μf A = -10^8,

then
$$\left[\frac{de_o}{dt}\right]_{max} = -0.1 \text{ mV/sec} .$$

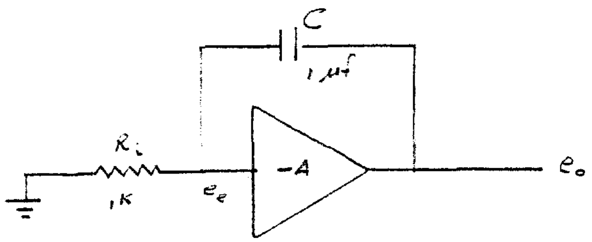


Fig. 3.

The smaller the value of R_i and/or C are the bigger $\frac{de_o}{dt}$ will be. Furthermore $\frac{de_o}{dT}$ also has to be considered.

4. <u>The thermal emf</u>. It is very obvious that the thermal emf is very small. Only its integration with respect to time introduces a significant error. That is only the emf (e_{th}) in the input circuit should be considered. The equation is

$$\frac{d}{dt} (e_o) = \frac{e_{th}}{R_i C} .$$

This indicates the fact that a small $R_i C$ time constant will enlarge the effect of e_{th}.

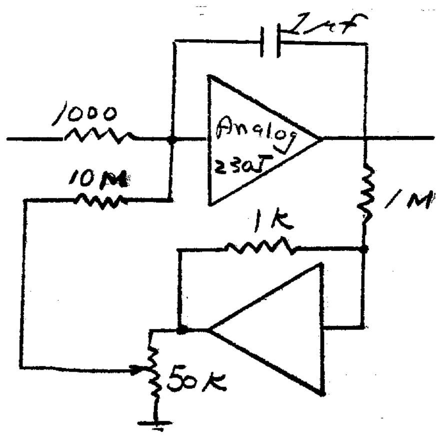
We can summarize the above discussion into the following equation:

$$\frac{\partial}{\partial t} (e_o) = K_1 + K_2 \ e_o ,$$

where K_1, K_2 are functions of T. Under a steady environmental condition and after the integrator reaches an equilibrium state, K_1, K_2 can be considered as constants. The first term can be compensated by voltage or current method (Ref. 3) and the second term can be taken care of by giving a slight positive feedback circuit arrangement. The suggested circuits are shown in Figs. 4 and 5 respectively. An accuracy of 0.05% can be achieved for the present circuit. A set of detailed test data will help further the quantitative analysis.

Fig. 4. (a) Current compensating circuit

(b) Voltage compensating circuit



value for 0 ~ 10% feedback and 10 second integration time constant.

Fig. 5. Positive feedback circuit.

III. DISCUSSION ON PRESENT INTEGRATORS

For a period of months, we have tested operational amplifiers of different types and manufacturers (Table I). The main interest was how well the drift can be controlled if the integration time constant is 1 msec and the source impedance is very low. We also reviewed the stability, signal-to-noise ratio, and reliability. Philbrick model 656 operational amplifier had several breakdowns during testing; others have been rejected because of oscillation, noise and/or unstable drift control.

Two different types of circuits have been built--the integrator and the integrator with a variable-gain amplifier. The circuits and their physical layouts are shown in Figs. 6, 7, 8 and 9. In the circuit, Analog 230J operational amplifiers were used, which can be replaced by Analog 230L if a better temperature performance is required. The feedback capacitor of the integrator is 100 volt, 1 μf polystyrene dielectric capacitor having insulation resistance greater than 10^6 megaohm, dielectric absorption 0.02% and temperature coefficient -120 ppm/ °C. An electronic (FET) switch, Burr Brown 9859/15, was used to initialize the integrator. This can be operated either by a push button switch or a plus 5 volt pulse such as the DTL logic output from the interface of Varian 620/I computer. The reset time constant is 1.1 millisecond. The accuracy and linearity of the integrator is within ±0.05% (Table II). The drift can be adjusted to 0.1 mV/sec.

IV. ACKNOWLEDGMENT

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REFERENCES

[1] L. J. Laslett, Some Aspects of Search Coil Design, July 5, 1954.

[2] G. V. Lago and L. M. Benningfield, Control System Theory, 1962.

[3] A Selection Handbook and Catalog Guide to Operational Amplifiers, Analog Devices, Inc.

Table I. Operational Amplifiers Reviewed.

Manufacturer	Model Number
Philbrick	656
Burr Brown	1503
Keithley	302
Fairchild	ADO 26B
Melcor	1626
	1680
Zeltex	148
	14508
Analog	210
	203
	230J
	230L

Table II. Integrator No. 1 Test Data.

$c = 1\ \mu f$
$R_i = 51$ kohm

Magnetic flux reading (B) from NMR in kG	Integrator reading (volt) from DVM	Ratio B/V
22.5	9.2411	2.4347
20.0	8.2160	2.4342
15.0	6.1636	2.4336
10.0	4.1088	2.4338
5.0	2.0552	2.4328

$c = 1\ \mu f$
$R_i = 300$ kohm

B in kG	V in volt	Ratio B/V
22.5	1.5650	14.3769
20.0	1.3920	14.3673
15.0	1.0443	14.3636
10.0	0.6962	14.3636
5.0	0.3480	14.3657

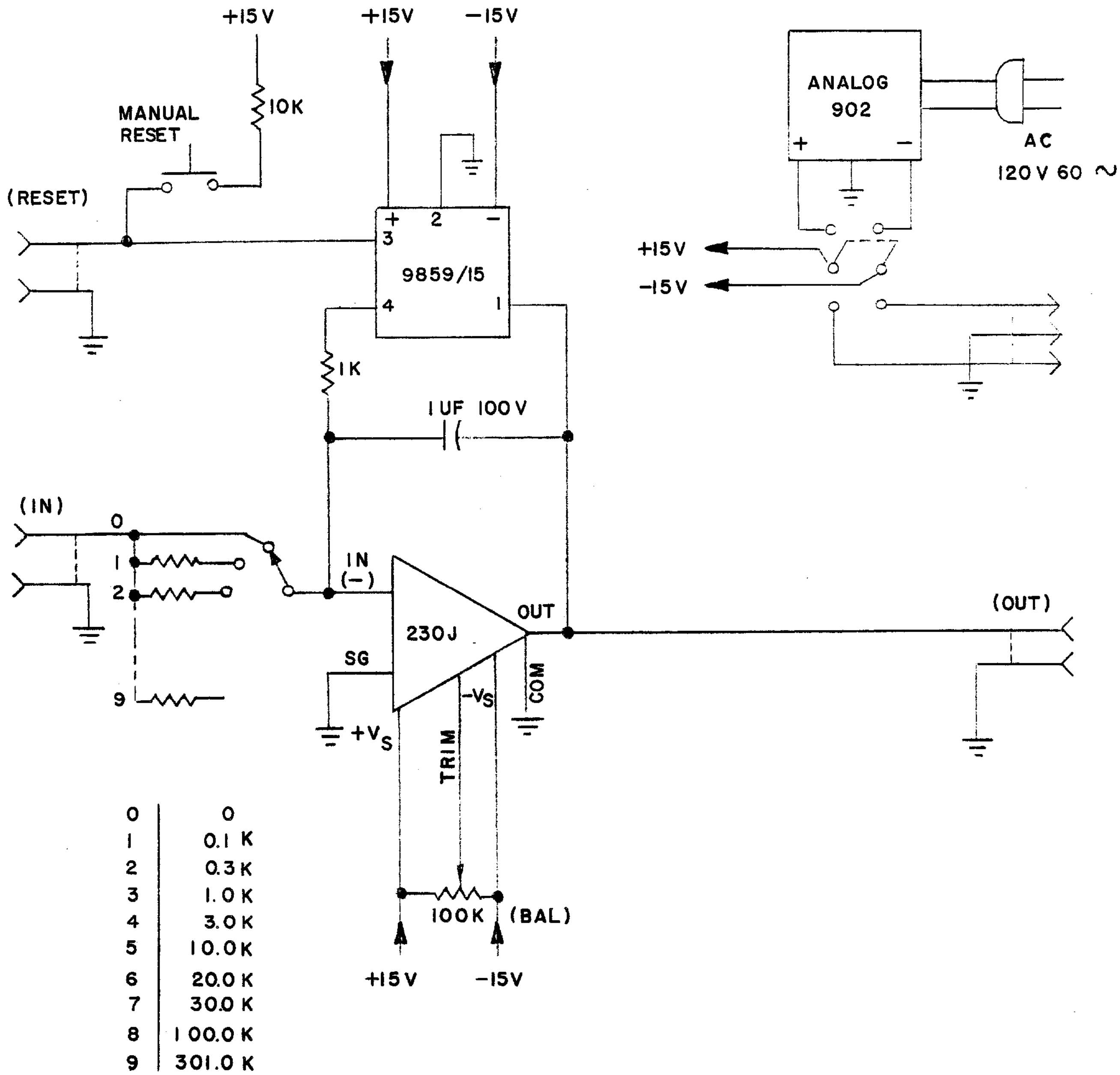

Fig. 6. Circuit diagram of the integrator.

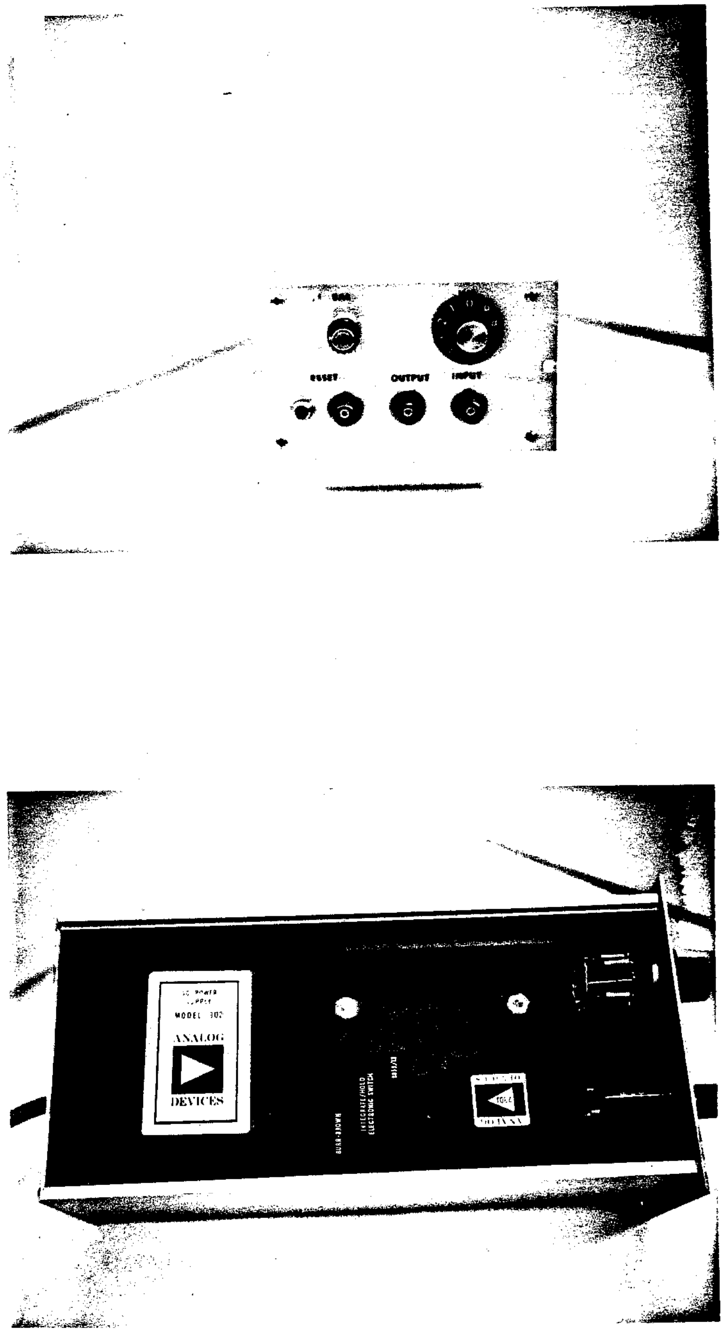

Fig. 7. Two sides view of the integrator.

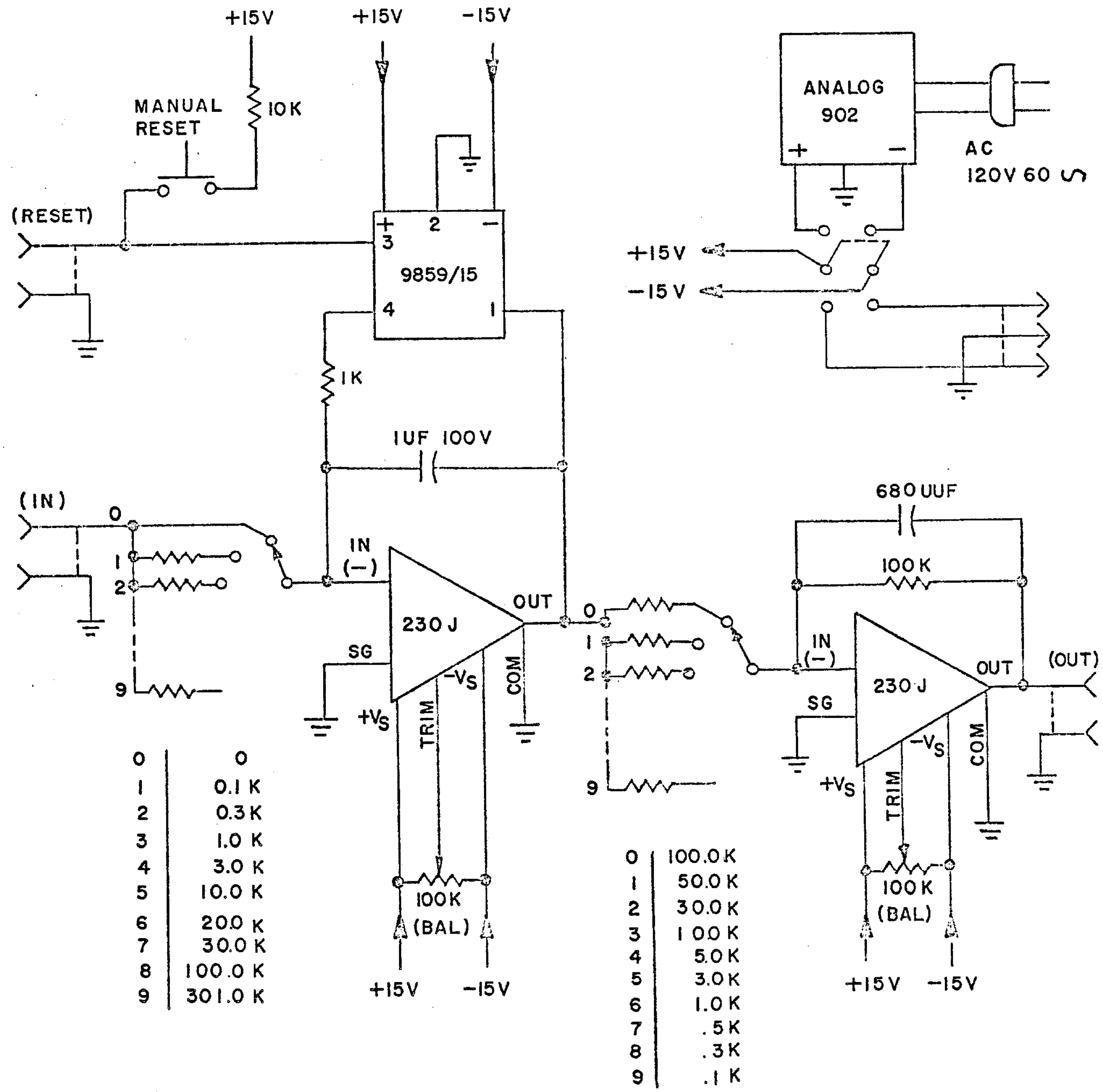

Fig. 8. Circuit diagram of the integrator with amplifier.

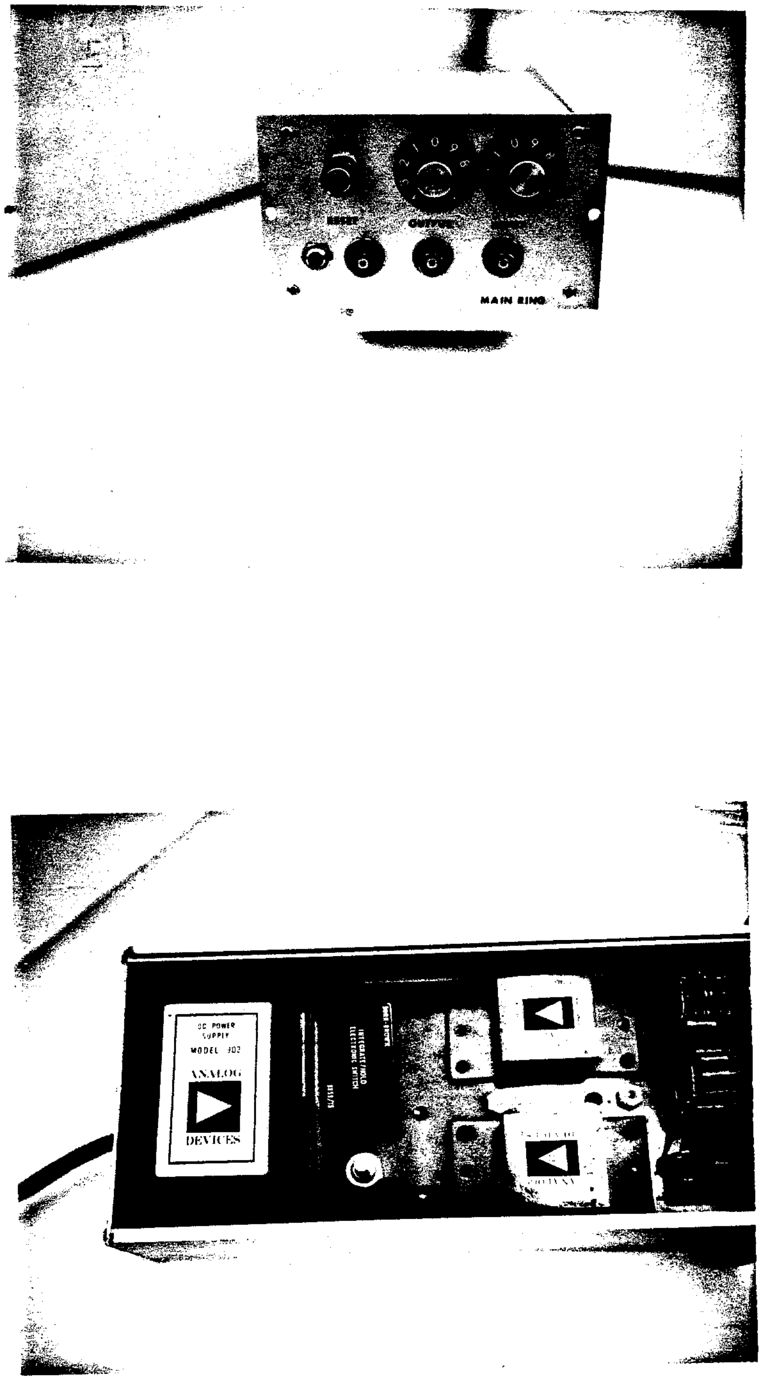

Fig. 9. Two sides view of the integrator with amplifier.